



Study Guide

Frequency Tables

Members of a seventh-grade class were surveyed to determine when to hold the winter dance. The results are shown at the right. On what date should the dance be held?

Explore What do you know?
 You know how each person responded.

 What are you trying to find?
 You are deciding on what date to hold the dance.

Plan Make a frequency table.

Solve The frequency table shows that the greatest number of people want to hold the dance on December 9.

Examine Since December 9 received the greatest number of votes, the winter dance should be held on December 9.

Dates Reported			
Dec. 9	Dec. 2	Dec. 9	Dec. 8
Dec. 2	Dec. 9	Dec. 8	Dec. 9
Dec. 9	Dec. 2	Dec. 9	Dec. 1
Dec. 2	Dec. 9	Dec. 9	Dec. 9
Dec. 8	Dec. 2	Dec. 9	Dec. 1
Dec. 2	Dec. 9	Dec. 2	Dec. 2
Dec. 8	Dec. 9		

Date	Tally	Frequency
Dec. 1		2
Dec. 2		8
Dec. 8		4
Dec. 9		12

Solve.

- Julia asked 12 people she bicycles with how many miles they rode their bicycles last week. She recorded the data in the table at the right.
 - What was the greatest distance ridden?
 - What was the least distance?
 - What was the range of the distances?
 - Choose an appropriate scale and interval.
- Twenty-one people were asked to name the state in which they were born. The data are shown below.

Miles Bicycled			
Dave	47.8	Gloria	56.9
Mary	19.9	Izumi	82.1
Lung	66.7	Monty	36.8
Iris	71.2	Kara	61.0
Cruz	56.0	Toshi	76.4
Leon	45.3	Burt	17.5

NY TX PA NY NY TX CA
 NY CA CA CA NY PA PA
 NY NY PA NY NY CA TX

- Make a frequency table for the data.
- In which state were the fewest people surveyed born?
- How many of the people surveyed were born in California?

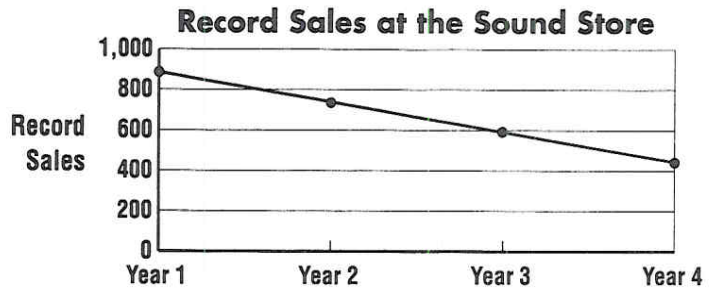
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Making Predictions

Graphs can be used to make predictions.

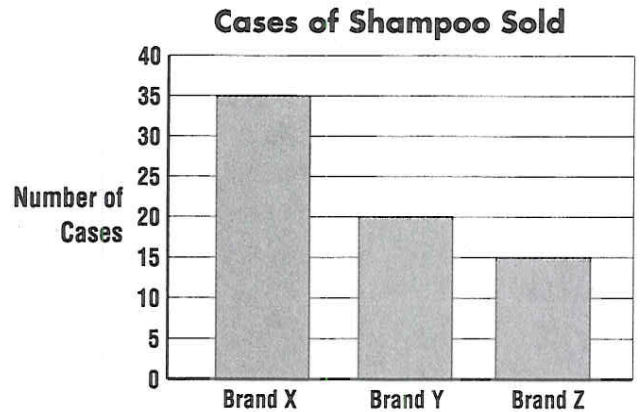
Example If the trend in the graph continues, predict about how many records the Sound Store will sell their fifth year in business.



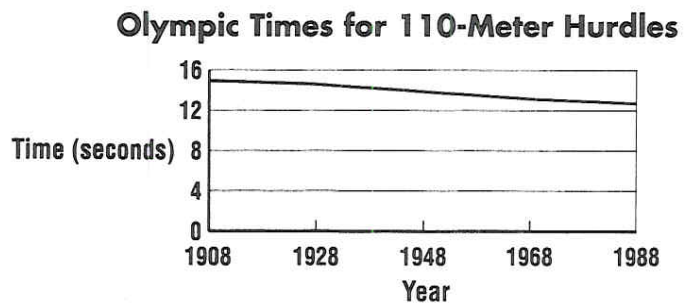
Each year about 150 fewer records have been sold. If the trend continues, the Sound Store will sell about 300 records in the fifth year.

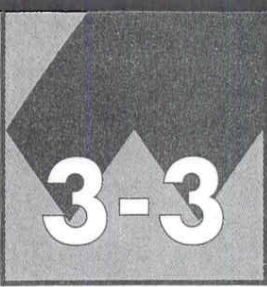
Solve.

1. A store owner is deciding how much of each brand of shampoo to order. She made a graph to show sales for the previous month. Based on the graph, of which brand should she order the most?



2. Predict the time for the 110-meter hurdles in the 2008 Olympics.



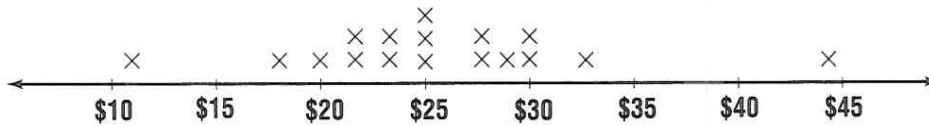


Study Guide

Line Plots

Darrell surveyed some kennels to find the cost of grooming his dog. The prices given were: \$25.00, \$27.00, \$32.00, \$22.00, \$43.00, \$28.00, \$18.00, \$24.00, \$25.00, \$27.00, \$30.00, \$24.00, \$22.00, \$30.00, \$12.00, \$25.00, and \$20.00.

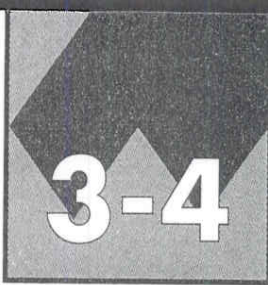
Darrell made a line plot to organize the data on a number line. First he found the range of the data: $\$43.00 - \$12.00 = \$31.00$. He chose a scale of \$10.00 to \$45.00 to include all of the data and an interval of \$5.00 to separate the data into 7 sections. He drew an \times to represent each price. For prices between marked intervals he estimated to position the \times .



The data are grouped or **clustered** between \$20 and \$30.

Make a line plot for each set of data.

- 560, 790, 800, 850, 350, 760, 810, 650, 850, 790, 690, 600
- 1,750, 2,000, 2,450, 1,900, 1,950, 1,900, 1,900, 1,900, 1,800, 2,100, 2,000, 1,800
- 7.1, 7.7, 7.8, 8.2, 8.4, 7.5, 7.8, 8.0, 8.3, 8.2, 8.4, 7.6, 8.0



Study Guide

Mean, Median, and Mode

To find the **mean**, or arithmetic average, of a set of numbers, find the sum of the numbers and divide by the number of items in the set.

To find the **median** of a set of numbers, arrange the numbers in order from least to greatest and find the middle number.

To find the **mode** of a set of numbers, find the number or item that appears most often.

Length of Nine Ladybugs (in inches)		
0.30	0.28	0.34
0.32	0.30	0.31
0.34	0.34	0.30

Example Find the mean, median, and mode of the ladybug lengths.

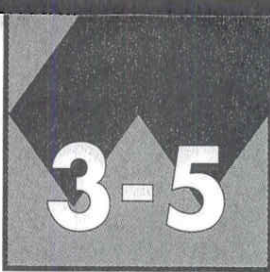
$$\text{mean} \quad \frac{0.30 + 0.28 + 0.34 + 0.32 + 0.30 + 0.31 + 0.34 + 0.34 + 0.30}{9} \approx 0.31$$

mode There are two modes for the data, 0.30 and 0.34.

median 0.28, 0.30, 0.30, 0.30, 0.31, 0.32, 0.34, 0.34, 0.34,
↑
median

Find the mean, mode(s), and median for each set of data.

- 6, 3, 7, 1, 8, 4, 8, 9, 4
- 5.6, 3.2, 7.1, 7.7, 9.0, 6.0, 5.3, 3.2, 4.2
- 70, 55, 42, 31, 78, 93, 54, 75, 35, 41, 64
- 2,300, 2,350, 2,240, 2,500, 2,300
- 21, 56, 34, 27, 42, 21, 77, 41, 77
- 450, 370, 190, 220, 540, 560, 270, 110, 230



Study Guide

Stem-and-Leaf Plots

The table below shows the number of home runs Hank Aaron hit by year.

Home Runs Hit by Hank Aaron												
Year	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
Home Runs	13	27	26	44	30	39	40	35	45	44	24	32
Year	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	
Home Runs	44	39	29	44	38	47	34	40	20	12	10	

A stem-and-leaf plot of the data is shown below.

The tens digits are the *stems*.

The ones digits are the *leaves*.

They are arranged in each row from least to greatest.

4 | 0 means 40.

Stem	Leaf
1	0 2 3
2	0 4 6 7 9
3	0 2 4 5 8 9 9
4	0 0 4 4 4 4 5 7

Make a stem-and-leaf plot for each set of data.

1. 89, 54, 67, 78, 65, 89, 57, 87, 75, 59, 65, 72, 59, 60, 73, 65

2. 85, 124, 90, 113, 107, 94, 88, 114, 106, 109, 110, 117, 100, 101, 119

Refer to the stem-and-leaf plot for Hank Aaron's home runs.

- What was the greatest number of home runs in a year? What was the fewest home runs in a year?
- In how many years did Hank Aaron hit 30 or more home runs?
- What is the mode for the data?
- What is the median for the data?

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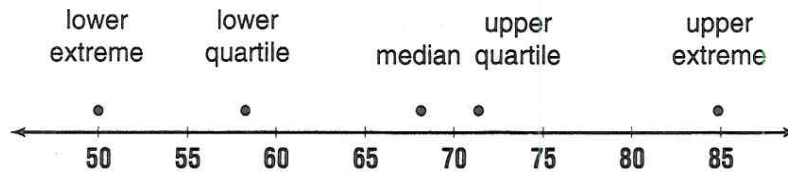
Box-and-Whisker Plots

A **box-and-whisker plot** summarizes data using the median, the upper and lower quartiles, and the extreme (highest and lowest) values.

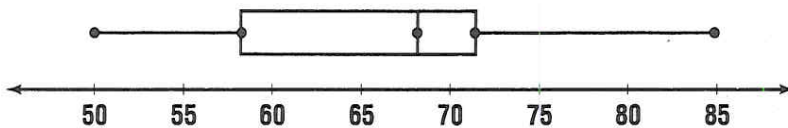
Example Draw a box-and-whisker plot to show these data.
 50 53 57 60 65 66 68 70 71 71 71 71 85

On a number line, graph the following points.

lower extreme: 50 upper extreme: 85 median: 68
 upper quartile: 71 lower quartile: 58.5

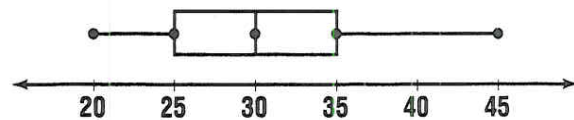


Draw a box around the quartile values.
 Draw a vertical line through the median.
 Extend whiskers from the quartiles to the upper and lower extremes.



Use the box-and-whisker plot below to answer each question.

1. What is the median?
2. What are the upper and lower extremes?
3. What is the interquartile range?
4. Draw a box-and-whisker plot for the data in the stem-and-leaf plot.



Stem	Leaf
3	0 5 6
4	0 2 6 7
5	1 3 8 9 9
6	0 2 4

$3/5 = 35$

Study Guide

Misleading Statistics

Don't be misled by statistics. Choose the best measure of central tendency to describe the data.

Measure of Central Tendency	When to Use
mean	when no numbers are much greater or much less than the rest
mode	when the most frequently occurring number is needed
median	when there are numbers that are much greater or much less than the rest

Example A car dealer advertised that the average savings on a new car purchased from her was \$1,500.

Number of Sales	Amount of Savings
2	\$5,000
1	\$3,500
9	\$500

The mean of the data in the table is \$1,500. The median and the mode are both \$500. \$500 is more representative of the savings you might expect if you buy a car from this dealer.

The members of a health club compiled the table below. It shows how many laps various swimmers completed.

- Find the mean, mode, and median of the data.
- Which measure is least descriptive?
- Which number most accurately describes the data?

Number of Laps	People
10	9
15	7
20	5
25	10

Study Guide

Divisibility Patterns

The following rules will help you determine if a number is divisible by 2, 3, 4, 5, 6, 9, or 10.

A number is divisible by:

- 2 if the digit in the ones place is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the digit in the ones place is 0 or 5.
- 6 if the number is divisible by 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the digit in the ones place is 0.

Example Determine whether 2,346 is divisible by 2, 3, 4, 5, 6, 9, or 10.

- 2: Yes; the ones digit, 6, is even.
- 3: Yes; the sum of the digits, $2 + 3 + 4 + 6 = 15$, is divisible by 3.
- 4: No; the number formed by the last two digits, 46, is not divisible by 4.
- 5: No; the ones digit is not 0 or 5.
- 6: Yes; the number is divisible by 2 and 3.
- 9: No; the sum of the digits, 15, is not divisible by 9.
- 10: No; the ones digit, 6, is not 0.
- 2,346 is divisible by 2, 3, and 6.

Determine whether the first number is divisible by the second number.

- | | | | |
|-------------|-------------|--------------|-------------|
| 1. 65; 5 | 2. 2,641; 3 | 3. 6,780; 10 | 4. 4,185; 9 |
| 5. 4,889; 2 | 6. 8,826; 4 | 7. 60,003; 6 | 8. 642; 4 |

Determine whether each number is divisible by 2, 3, 4, 5, 6, 9, or 10.

- | | | | |
|--------|-----------|-----------|---------|
| 9. 660 | 10. 5,025 | 11. 5,091 | 12. 356 |
|--------|-----------|-----------|---------|

Study Guide

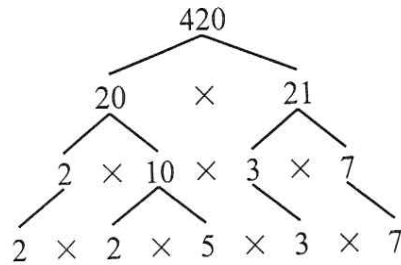
Prime Factorization

A **prime number** is a whole number greater than 1 that has exactly two factors, 1 and itself.

- Examples** 7 factors: 1, 7
 23 factors: 1, 23

A **composite number** is a whole number greater than 1 that has more than two factors. Every composite number can be written as the product of prime numbers. This is called the **prime factorization** of the number.

Example Write the prime factorization of 420.



*Write 420 as the product of two factors.
 Keep factoring until all of the factors are prime numbers.*

The prime factorization of 420 is $2 \times 2 \times 3 \times 5 \times 7$, or $2^2 \times 3 \times 5 \times 7$.

Determine whether each number is composite or prime.

- | | | | |
|-------|-------|--------|-------|
| 1. 34 | 2. 77 | 3. 37 | 4. 89 |
| 5. 69 | 6. 67 | 7. 123 | 8. 71 |

Write the prime factorization of each number.

- | | | | |
|--------|---------|-----------|-----------|
| 9. 490 | 10. 225 | 11. 1,155 | 12. 1,105 |
|--------|---------|-----------|-----------|

Study Guide

Integration: Patterns and Functions
Sequences

A **sequence** of numbers is a list in a specific order. The numbers in the sequence are called **terms**.

A sequence is an **arithmetic sequence** if you can always find the next term by adding the same number to the previous term.

Examples 1 $7, \overset{\curvearrowright}{+4} 11, \overset{\curvearrowright}{+4} 15, \overset{\curvearrowright}{+4} 19, \overset{\curvearrowright}{+4} 23, \dots$ *The next term can be found by adding 4 to the previous term.*

The next three terms are 27, 31, and 35.

2 $76, \overset{\curvearrowright}{-3} 73, \overset{\curvearrowright}{-3} 70, \overset{\curvearrowright}{-3} 67, \overset{\curvearrowright}{-3} 64, \dots$ *The next term can be found by subtracting 3 from the previous term.*

The next three terms are 61, 58, and 55.

1 A sequence is a **geometric sequence** if you can find the next term by multiplying the previous term by the same number.

Example 3 $576, \overset{\curvearrowright}{\times 0.5} 288, \overset{\curvearrowright}{\times 0.5} 144, \overset{\curvearrowright}{\times 0.5} 72, \dots$ *The next term can be found by multiplying the previous term by 0.5.*

The next three terms are 36, 18, and 9.

A sequence may be neither arithmetic nor geometric.

Example 4 $3, \overset{\curvearrowright}{+1} 4, \overset{\curvearrowright}{+2} 6, \overset{\curvearrowright}{+3} 9, \overset{\curvearrowright}{+4} 13, \dots$ *The next term can be found by adding one more than the number added to the previous term.*

The next three terms are 18, 24, and 31.

Identify each sequence as arithmetic, geometric, or neither.
Then find the next three terms.

1. 89, 86, 83, 80, ...

2. 5, 25, 125, 625, ...

3. 7, 12, 17, 22, ...

4. 78, 75, 77, 74, 76, ...

5. 64, 32, 16, 8, ...

6. 90, 85, 79, 72, ...

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Study Guide

Greatest Common Factor

The **greatest common factor (GCF)** of two or more numbers is the greatest number that is a factor of each number. One way to find the GCF is to list the factors of each number and then choose the greatest of the common factors.

Example Find the GCF of 72 and 108.

factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

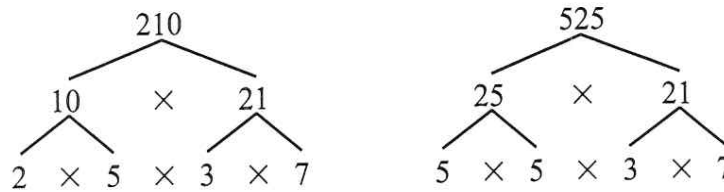
factors of 108: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

common factors: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF of 72 and 108 is 36.

Another way to find the GCF is to write the prime factorization of each number. Then identify all common prime factors and find their product.

Example Find the GCF of 210 and 525.



common prime factors: 3, 5, 7

The GCF of 210 and 525 is $3 \times 5 \times 7$ or 105.

Find the GCF of each set of numbers.

1. 18, 30

2. 60, 45

3. 24, 72

4. 32, 48

5. 100, 30

6. 54, 36

7. 120, 200

8. 81, 153

9. 77, 121

10. 60, 24, 72

11. 32, 48, 80

12. 90, 120, 180

Study Guide

Simplifying Fractions and Ratios

A fraction is in **simplest form** when the greatest common factor (GCF) of the numerator and the denominator is 1.

Example 1 Express $\frac{36}{54}$ in simplest form.

factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

The GCF of 36 and 54 is 18.

$$\frac{36}{54} = \frac{36 \div 18}{54 \div 18} = \frac{2}{3} \quad \text{Divide the numerator and denominator by the GCF.}$$

You can also express a ratio in simplest form.

Example 2 Express 28:63 in simplest form.

factors of 28: 1, 2, 4, 7, 14, 28

factors of 63: 1, 3, 7, 9, 21, 63

The GCF of 28 and 63 is 7.

$$\frac{28}{63} = \frac{28 \div 7}{63 \div 7} = \frac{4}{9} \quad \text{Divide the numerator and denominator by the GCF.}$$

Express each fraction or ratio in simplest form.

1. $\frac{30}{72}$

2. 45:60

3. $\frac{68}{84}$

4. 54:66

5. 56:64

6. $\frac{17}{119}$

7. $\frac{60}{75}$

8. $\frac{75}{375}$

9. 36:48

10. 33:132

11. $\frac{450}{750}$

12. 25:125

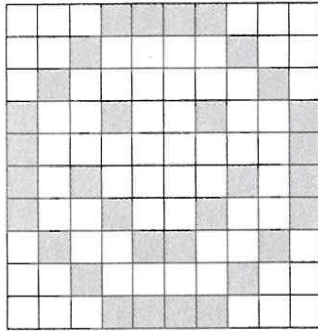
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Study Guide

Ratios and Percents

A percent is a ratio that compares a number to 100.

Examples 1 Write a percent to represent the shaded area of the model.



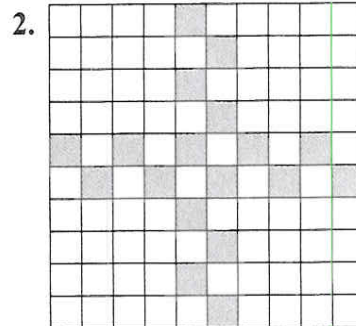
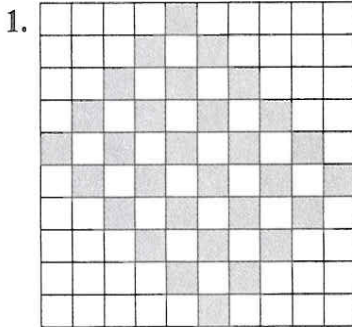
32 out of 100 squares are shaded.

$$\frac{32}{100} = 32\%$$

2 Express 28:63 as a percent.

$$\frac{28}{63} = 44.\overline{4}\%$$

Write a percent to represent the shaded area.



Express each ratio as a percent.

3. $\frac{72}{100}$

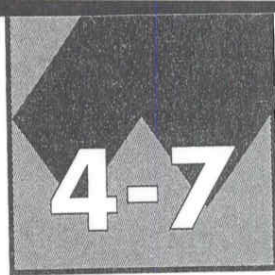
4. 45:100

5. 68 people out of 100

6. 66:100

7. 7 to 100

8. 17 in 100



Study Guide

Fractions, Decimals, and Percents

A fraction can be expressed as a percent by finding an equivalent fraction with a denominator of 100.

Example 1 Express $\frac{19}{20}$ as a percent.

$$\frac{19}{20} = \frac{95}{100} = 95\%$$

(Note: Arrows in the original image indicate that both the numerator and denominator of $\frac{19}{20}$ are multiplied by 5 to get $\frac{95}{100}$.)

Since $100 \div 20 = 5$, multiply the numerator and denominator by 5.

When an equivalent fraction cannot easily be found, express the fraction as a decimal first, and then as a percent.

Example 2 Express $\frac{5}{8}$ as a percent.

$$5 \div 8 = 0.625 \text{ or } 62.5\%$$

You can also express decimals and percents as fractions.

Examples $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

$$18\% = \frac{18}{100}$$
$$= \frac{18 \div 2}{100 \div 2} \text{ or } \frac{9}{50}$$

Express each fraction as a percent.

1. $\frac{14}{25}$

2. $\frac{3}{4}$

3. $\frac{7}{8}$

4. $\frac{7}{10}$

5. $\frac{6}{50}$

6. $\frac{13}{20}$

Express each percent or decimal as a fraction in simplest form.

7. 20%

8. 0.60

9. 0.15

10. 72%

11. 54%

12. 0.22

Study Guide

Integration: Probability Simple Events

If you roll a cube with the numbers 1 through 6 on the faces, there are six possible outcomes: 1, 2, 3, 4, 5, and 6. Each of the outcomes is equally likely to occur. A particular outcome, such as rolling a 5, is an event. Probability is the chance that the event will occur.

$$\text{Probability} = \frac{\text{number of ways an event can occur}}{\text{number of possible outcomes}}$$

Rolling a 5 can occur 1 way out of 6 possible outcomes. So, $P(5) = \frac{1}{6}$.

Example Table tennis balls numbered 1 through 25 are placed in a box and one is drawn at random. Find each probability.

probability of drawing a 12:

$$P(12) = \frac{\text{number of ways 12 can occur}}{\text{number of possible outcomes}} = \frac{1}{25}$$

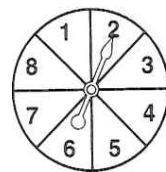
probability of drawing an odd number:

The odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, and 25. There are thirteen balls with an odd number.

$$P(\text{odd}) = \frac{\text{number of ways an odd number can occur}}{\text{number of possible outcomes}} = \frac{13}{25}$$

The spinner shown is equally likely to stop on each of the regions. Find the probability that the spinner will stop on each of the following.

- | | |
|-------------------------|--------------------|
| 1. a number less than 5 | 2. an even number |
| 3. a prime number | 4. a multiple of 4 |
| 5. a factor of 8 | 6. 7 |



A drawer contains 4 blue socks, 8 black socks, and 10 white socks. If one sock is taken out of the drawer without looking, find the probability that each of the following will be drawn. Express each ratio as a fraction in simplest form.

- | | |
|-----------------------------|----------------------------|
| 7. a blue sock | 8. a black sock |
| 9. a white sock | 10. a blue or a black sock |
| 11. a black or a white sock | 12. a blue or a white sock |



Study Guide

Least Common Multiple

A **multiple** of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the **least common multiple (LCM)** of the numbers.

Example 1 Find the LCM of 15 and 20.

multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, . . .

multiples of 20: 20, 40, 60, 80, 100, 120, 140, . . .

The LCM of 15 and 20 is 60.

Prime factorization can also be used to find the LCM.

Example 2 Find the LCM of 8, 12, and 18.

$$8 = 2 \times 2 \times 2$$

Find the prime factors of each number.

$$12 = 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$\begin{array}{ccc} 2 & 2 & 3 \end{array}$$

Find the common factors.

$$2 \times 2 \times 2 \times 3 \times 3 = 72$$

Multiply the common factors and any other factors.

The LCM of 8, 12, and 18 is 72.

Find the LCM of each set of numbers.

1. 12, 16

2. 15, 24

3. 7, 9

4. 8, 10

5. 20, 50

6. 18, 27

7. 30, 21

8. 12, 18

9. 6, 10, 15

10. 3, 7, 10

11. 2, 16, 24

12. 7, 8, 14

Study Guide

Comparing and Ordering Fractions

To compare fractions, rewrite them so they have the same denominator. The **least common denominator (LCD)** of two fractions is the least common multiple of their denominators.

Example 1 Which fraction is greater, $\frac{5}{6}$ or $\frac{3}{4}$?

Find the LCD by listing the multiples of each denominator.

multiples of 6: 6, 12, 18, 24, 30, 36, ...

multiples of 4: 4, 8, 12, 16, 20, 24, ...

The LCM of 6 and 4 is 12. So, the LCD of $\frac{5}{6}$ and $\frac{3}{4}$ is 12.

Write $\frac{5}{6}$ and $\frac{3}{4}$ as fractions with a denominator of 12.

$$\begin{array}{c} \times 2 \\ \curvearrowright \\ \frac{5}{6} = \frac{10}{12} \\ \curvearrowleft \\ \times 2 \end{array}$$

$$\begin{array}{c} \times 3 \\ \curvearrowright \\ \frac{3}{4} = \frac{9}{12} \\ \curvearrowleft \\ \times 3 \end{array}$$

$$\frac{10}{12} > \frac{9}{12}, \text{ so } \frac{5}{6} > \frac{3}{4}.$$

Another way to compare fractions is to express them as decimals. Then compare the decimals.

Example 2 Which fraction is greater, $\frac{7}{9}$ or $\frac{3}{4}$?

Express each fraction as a decimal. Then compare.

$$7 \div 9 = 0.\overline{7} \quad 3 \div 4 = 0.75 \quad 0.\overline{7} > 0.75, \text{ so } \frac{7}{9} > \frac{3}{4}.$$

Find the LCD for each pair of fractions.

1. $\frac{1}{2}, \frac{1}{3}$

2. $\frac{3}{4}, \frac{1}{8}$

3. $\frac{5}{9}, \frac{1}{2}$

4. $\frac{2}{3}, \frac{3}{7}$

5. $\frac{5}{9}, \frac{5}{6}$

6. $\frac{7}{8}, \frac{5}{12}$

7. $\frac{7}{10}, \frac{4}{5}$

8. $\frac{3}{4}, \frac{1}{2}$

Replace each \bigcirc with $<$, $>$, or $=$ to make a true sentence.

9. $\frac{1}{2} \bigcirc \frac{5}{9}$

10. $\frac{3}{4} \bigcirc \frac{7}{8}$

11. $\frac{5}{12} \bigcirc \frac{1}{2}$

12. $\frac{4}{5} \bigcirc \frac{7}{10}$